Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Advanced Real Analysis

Subject Code : 5SC03ARA1 **Branch: M.Sc. (Mathematics)**

Semester: 3 **Date: 25/02/2020** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Define: σ - finite measure **(01)**

b. What is the mean of Signed measure space? **(02)**

c. Define: Complete measure **(02)**

d. Define: Mutually singular measure **(02)**

Q-2 **Attempt all questions** **(14)**

a. Suppose υ_1 and υ_2 are two finite signed measure on $(X,\mathcal{A})$ then show that $\alpha\upsilon_1+\beta\upsilon_2$ is also signed measure, for any $\alpha,\beta\in R$. Further prove the following: i) $|\alpha\upsilon_1|=|\alpha||\upsilon_1|$ and ii) $|\upsilon_1+\upsilon_2|\le|\upsilon_1|+|\upsilon_2|$. **(06)**

b. State and prove Beppo-levi's theorem. **(04)**

c. If $E_1,E_2\in\mathcal{A}$ then show that $\mu(E_1\Delta E_2)=0\Rightarrow\mu(E_1)=\mu(E_2)$. Moreover if μ is complete and $E_1\in\mathcal{A}$ with $\mu(E_1\Delta E_2)=0$ then $E_2\in\mathcal{A}$ **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Lusin's theorem. **(08)**

b. State and prove Lebesgue Dominated Convergence theorem. **(06)**

Q-3 **Attempt all questions** **(14)**

a. State and prove Hahn-Decomposition theorem. **(07)**

b. Let E be measurable set with $0<\nu(E)<\infty$ then E contains a positive set A with $\nu(A)>0$. **(05)**

	c.	Define: Absolutely continuous measure	**(02)**

OR

Q-3		**Attempt all questions**	**(14)**
	a.	State and prove Jordan Decomposition theorem.	**(08)**
	b.	Define: Positive set and prove that countable union of positive set is positive set.	**(04)**
	c.	Let $(X,\mathcal{A})$ be a measurable space if $\lambda \& \mu$ are two measure on $(X,\mathcal{A})$ with $\lambda \perp \mu$ and $\lambda << \mu$ then $\lambda = 0$.	**(02)**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Define: Product measure space	**(01)**
	b.	State Tonelli's theorem.	**(02)**
	c.	What is the mean of conjugate pair of real numbers?	**(02)**
	d.	Suppose $f \in L^p(\mu)$ and $f = g$ a.e. and μ be a complete measure then $g \in L^p(\mu)$.	**(02)**
Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Radon-Nikodym theorem.	**(10)**
	b.	Show that Holder's inequality is an equality if $\alpha\|f\|^p = \beta\|g\|^q$ a.e. on *X*, for some $\alpha, \beta \in R - \{0\}$.	**(04)**

OR

Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Riesz-Ficher's theorem.	**(08)**
	b.	Prove that *X* is Banach space if and only if every absolutely summable series is summable.	**(06)**
Q-6		**Attempt all questions**	**(14)**
	a.	Prove that L^p spaces are complete spaces.	**(08)**
	b.	Prove the set of all simple measurable function f vanishing outside a set of finite measure is dense in $L^p(\mu)$, where $1 \le p < \infty$.	**(06)**

OR

Q-6		**Attempt all Questions**	**(14)**
	a.	State and prove Caratheodory theorem.	**(09)**
	b.	What is the Riesz representation theorem? Explain it with proof.	**(05)**